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A HEURISTIC STRATEGY TO COMPUTE ENSEMBLES OF TRAJECTORIES FOR 3D LOW-COST EARTH-MOON TRANSFERS

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The problem of finding optimal trajectories is essential for modern space mission design. When considering multi-body gravitational dynamics and exploiting both low-thrust and high-thrust and alternative forms of propulsion such as solar sailing, sets of good initial guesses are fundamental for the convergence to local or global optimal solutions, using both direct or indirect methods available to solve the optimal control problem. This paper deals with obtaining preliminary trajectories that are designed to be good initial guesses as input to search optimal low-energy short-time Earth-Moon transfers with ballistic capture. A more realistic modelling is introduced, in which the restricted four-body system Sun-Earth-Moon-Spacecraft is decoupled in two patched planar Circular Restricted Three-Body Problems, taking into account the inclination of the orbital plane of the Moon with respect to the ecliptic. We present a heuristic strategy based on the hyperbolic invariant manifolds of the Lyapunov orbits around the Lagrangian points of the Earth-Moon system to obtain ballistic capture orbits around the Moon that fulfill specific mission requirements. Moreover, quasi-periodic orbits of the Sun-Earth system are exploited using a genetic algorithm to find optimal solutions with respect to total Δv, time of flight and altitude at departure. Finally, the procedure is illustrated and the full transfer trajectories assessed in view of relevant properties. The proposed methodology provides sets of low-cost and short-time initial guesses to serve as inputs to compute fully optimized three-dimensional solutions considering different propulsion technologies, such as low, high, and hybrid thrust, and/or using more realistic models.

I. INTRODUCTION

Direct transfers to the Moon and other planets are rarely an option nowadays due to their very high requirements in terms of total acceleration, or Δv, which implies in a high propellant mass fraction. Thus, the models employed in mission analysis have become more realistic in order to obtain low-cost solutions. In fact, multi-body gravitational dynamics has been used to enable orbits that do not exist in two-body dynamics. Moreover, the missions have increased in complexity, exploiting both low-thrust and high-thrust, including alternative forms of propulsion such as solar sailing.

The problem of finding optimal trajectories in a multi-body environment using modern forms of propulsion requires a combination of dynamical systems theory and global and local optimisation techniques. In this context, sets of good initial guesses are fundamental for the convergence to local or global optimal solutions, using both direct or indirect methods available to solve the optimal control problem.

In particular, in the case of Earth-Moon mission design, the Circular Restricted Three-Body Problem (CR3BP) provides the theoretical background for a class of low energy transfers that relies on the invariant manifolds associated to the equilibrium points of the Sun-Earth-Spacecraft (SE) and Earth-Moon-Spacecraft (EM) systems. Recent missions, such as NASA's GRAIL and CNSA's Chang'e-2, have successfully benefited from this model to reach and explore Earth's natural satellite.

It is possible to exploit the mathematical structures of the CR3BP by employing the *patched three-body approach*, an approximation in which the restricted four-body system Sun-Earth-Moon-Spacecraft is decoupled in two patched CR3BPs [1, 2].

The standard Earth-Moon transfers in the patched three-body approach are low-energy solutions formed by two arcs. The first arc is non-transit orbit associated to periodic solution around either L_1 or L_2 of the SE system, and the second arc is transit orbit associated to a period solution around L_2 of the EM system. Because these are manifold guided solutions, they require long transfer time, usually more than 100 days [3, 4, 5, 6].

Alternatively, there are short-transfer-time solutions that connect quasi-periodic orbits on two-dimensional tori of the SE system with L_1 or L_2 transit solutions of the EM system, such that the transfer time is reduced to about 10 days, while still providing ballistic capture by the Moon [7, 8].

In this paper, we discuss a heuristic strategy to obtain ensembles of ballistic capture orbits around the Moon. This, combined with a genetic algorithm to survey quasi-periodic solutions of the Sun-Earth system, produces good initial guesses aiming optimized solutions, with low, high, and hybrid thrust. The resulting Earth-Moon transfers are low-cost short-transfer time solutions with long time of permanence around the Moon.

We introduce a more realistic modelling which still takes advantage of the patched three-body approximation but takes into account the inclination of the orbital plane of the Moon with respect to the ecliptic. Then we describe a systematic design strategy that can be decomposed in two separate parts. The first part is based on the hyperbolic invariant structures associated to the Lagrangian points of the EM system, and aims to select ballistic capture orbits around the Moon that fulfill specific mission requirements. The second part consists in exploiting quasi-periodic orbits of the SE system by using a genetic algorithm to find optimal solutions with respect to total Δv, time of flight and altitude at departure. The full two-arc solutions correspond to initial guesses to be considered afterwards in the computation of fully optimized solutions considering different propulsion technologies, such as low, high, and hybrid thrust, and/or using more realistic models.

The paper is organized as follows: Section II introduces the patched three-body approximation with tilted planes and briefly recalls the dynamical model. Then, Section III presents a systematic procedure to compute Earth-Moon transfers within this approximation by selecting ballistic capture trajectories in the Earth-Moon system, and translating them to the SE system, to investigate the cost of the patching possibilities. The results of applying the methodology of Section III are presented in Section IV, where the steps of the procedure are illustrated and full transfer trajectories are obtained. Finally, final remarks are made in Section V.

II. MATHEMATICAL BACKGROUND

In the context of Earth-Moon transfers, the usual patched three-body approximation considers the EM system and the SE system to be coplanar. So, to a first approximation, the Sun-Earth-Moon-Sc is modeled as two coupled planar CR3BPs with all the primaries in the same plane.

However, the orbit of the Moon around the Earth is inclined with respect to the plane of the mean motion of the Earth around the Sun. So, in this work, we introduce an alternative modelling in which the two planar CR3BPs are inclined with respect to each other. As the dynamics of both systems are assumed to be planar, the patching point of the complete Earth-Moon transfer is on the line of the nodes, that is at the intersection of the orbital planes of each pair of primaries.

Although the plane of the lunar orbit precesses in space, making a complete revolution with respect to the vernal equinox every 18.612958 years, in this work, because the duration of a transfer orbit will not exceed a few days, it suffices to assume that the direction of the line of nodes is constant with respect to the inertial SE system. Moreover, the axes of the SE synodical system and of the SE inertial system are assumed to coincide at the origin of time.

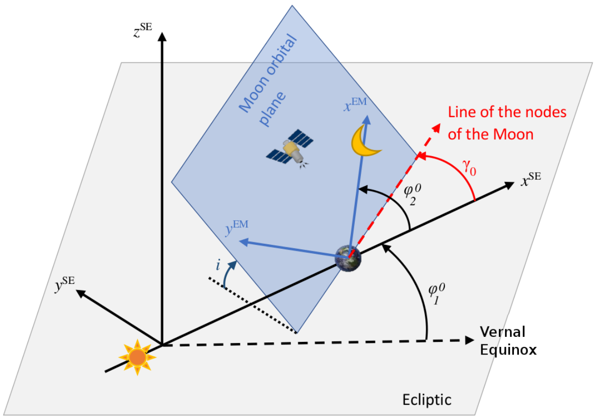

Figure 1: Schematic representation of the EM and SE systems with tilted planes.

Figure 1 illustrates the setup of the patched approximation with tilted planes. The direction of the line of nodes can be chosen based on ephemeris data to have a date with a convenient configuration of the primaries and is given by $\hat{N} = (\cos(\gamma_0), \sin(\gamma_0), 0)$, where γ_0, the angle that departs from the axis connecting the Sun and the Earth at the initial instant of time. Moreover, the angle φ_1^0 gives the position of the x-axis of the SE synodical system at the origin of time with respect to the vernal equinox.

To connect the solution arcs of both systems, we start with the solution arc in the EM planar barycentric synodic frame and apply a sequence of geometric transformations to go to the SE planar barycentric synodic reference frame. Also, an impulsive thrust, Δv_2, at the patching point is needed to close the energy gap between both systems.

Let t_{EM} and t_{SE}, respectively, be the time of flight in the EM reference frame and the time of flight in the SE reference frame. This sequence of transformations includes a clockwise rotation of $(t_{EM}+\varphi_2^0)$ around the EM z-axis, a translation of the origin from the EM barycenter to the Earth position, a rotation around $\hat{N}$ of the angle $i = 5.145°$, a scaling transformation from the units of the EM system to the units of the SE system, a translation of the origin from the Earth to the barycenter of the SE system, and finally a rotation around the SE z-axis of t_{SE}. For a given value of γ_0, the angle between the x-axis of the SE synodical system and the x-axis of the EM synodical system, φ_2^0, is computed to yield null z coordinate in the SE reference frame so that the patching point is at the line of the nodes. Also, to have purely planar in the SE system, the z-component of Δv_2, $\delta\dot{z}$, is computed such that the resulting SE state has null velocity in the z-direction.

Within each three-body system, the dynamics is ruled by the usual equations of motion of the planar CR3BP. So, in the synodic reference frame with dimensionless units, the motion of the spacecraft is given by

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \Omega_x, \\ \ddot{y} + 2\dot{x} &= \Omega_y, \end{aligned} \qquad [1]$$

where Ω is the effective potential given by

$$\Omega(x,y) = \frac{x^2+y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2}, \qquad [2]$$

with $r_1^2 = (x+\mu)^2 + y^2$ and $r_2^2 = (x-1+\mu)^2 + y^2$ being the square of the dimensionless distances from the spacecraft to the primaries P_1 and P_2, respectively, which are located at $(-\mu,0,0)$ and $(1-\mu,0,0)$. As usual, μ, is the normalized mass of P_2. For the EM and the SE systems μ is, respectively, $\mu_{EM} = 1.21506683 \times 10^{-2}$ and $\mu_{SE} = 3.03591 \times 10^{-6}$. In the case of μ_{SE}, the mass of the Moon is inclued in the normalization along with the mass of the Earth, so, in fact, the SE system corresponds to the Sun-(Earth-Moon) CR3BP.

Equation (1) has a first integral, called the Jacobi integral, which is given by

$$J(x,y,\dot{x},\dot{y}) = 2\Omega(x,y) - (\dot{x}^2+\dot{y}^2) = C, \qquad [3]$$

and five equilibrium points: L_1, L_2, and L_3, located on the x-axis, and L_4 and L_5, located at $(-\mu + 1/2, \mp\sqrt{3}/2)$.

III. A SYSTEMATIC PROCEDURE TO COMPUTE ENSEMBLES OF EARTH-MOON TRANSFERS

A systematic procedure to find full Earth-Moon transfers using patched three-body systems can be decomposed in two separate parts. The first part consists in an extensive analysis to seek of ballistic capture trajectories in the EM system. Then, a given capture solution can be translated into the SE system to investigate the patching possibilities and the cost of targeting the Earth backwards.

Figure 2 illustrates the setup of the numerical experiments in the EM system. We define a Poincaré section Σ_1 by $x = 0.75$, $\dot{x} > 0$, at the left side of L_1^{EM}, with $x_{L_1^{EM}} = 0.8369$, at a distance of 91,429 km from the Moon. For a given value of C, the smallest rectangle in the $(y,\dot{y})$ plane that contains the cut of the outer branch of the stable manifold, W_o^s, of $\Gamma(L_1^{EM})$ in Σ_1 is discretized in a grid of 500×500 points. Depending on the value of C, a given pair $(y,\dot{y})$ with constant $x = 0.75$ could render $\dot{x}^2 = 2\Omega(x,y) - C - \dot{y}^2 < 0$ so that this point does not correspond to a feasible initial condition. Each point in the grid that corresponds to a feasible initial condition that is evolved forward and classified into one of the following sets.

Set $\mathscr{G}$ (good): initial conditions of trajectories that cut the Poincaré section Σ_2 twice, with both cuts inside the lunar SOI, and with perilune between the first and second cuts with altitude between 100 km and 400 km.

Set $\mathscr{L}$ (low): initial conditions of trajectories that cut the Poincaré section Σ_2 twice, with both cuts inside the lunar SOI, and with perilune between the first and second cuts with altitude below 100 km.

Set $\mathscr{H}$ (high): initial conditions of trajectories that cut the Poincaré section Σ_2 twice, with both cuts inside the lunar SOI, and with perilune between the first and second cuts with altitude above 400 km.

Set $\mathscr{C}$ (collisional): initial conditions of trajectories that collide with the surface of the Moon (considering the lunar mean radius of 1738 km) before cutting Σ_2 twice, or before escaping the lunar SOI.

Set $\mathscr{O}$ (outside the lunar SOI): initial conditions of trajectories that cut Σ_2 outside the lunar SOI or that leave the lunar SOI before cutting Σ_2 twice.

The Poincaré section Σ_2 is given by $x = x_{EM}^{Moon} = 1 - \mu_{EM}$, $\dot{x} > 0$.

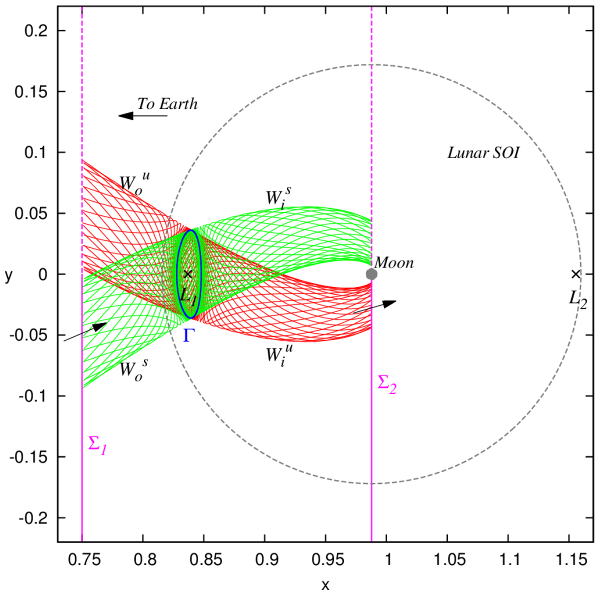

Figure 2: Setup of the numerical experiments of the EM system with the relevant dynamical invariant sets.

A variable step size Runge-Kutta-Fehlberg 7th-8th order solver [9] with relative error under 10^{-14} and absolute error under 10^{-15} is used to integrate the trajectories. For each numerical solution, the maximum integration time allowed is $t_f = 180$ days, but integration is terminated once the trajectory is classified.

Next, we consider the subset of initial conditions in Σ_1 which have trajectories with perilune altitude, h_m, between 90 and 200 km because this range of altitude corresponds to typical values in practical missions, that is, part of the initial conditions in $\mathscr{G}$ and $\mathscr{L}$. Each solution is integrated until one of the following conditions is satisfied:

(i) the spacecraft escapes the lunar region;
(ii) the spacecraft collides with the Moon;
(iii) final integration time, $t_f = 180$ days, is reached.

The initial conditions of trajectories that remain in the lunar region for long time, that is, that have large values of escape time, t_{esc}, are selected as suitable hyperbolic capture orbits. Once these orbits are obtained, we move to the second part of procedure to find full Earth-Moon transfer: the capture orbits are translated into the SE system to investigate the patching possibilities and the cost of targeting the Earth.

Consider the annular region around the Moon with inner radius of 70,000 km and outer radius of 91,429 km. This is an adequate region for patching the two systems because it is outside but near the lunar sphere of influence, SOI, whose boundary, as predicted by the Lagrange's SOI definition, is located at 66,190 km from the Moon [10]. The subset of points of each EM solution located within this region are assigned as candidate points for patching of the two three-body systems.

Let p_a^p and p_b^p denote the points along a given EM trajectory with time t_a^p and t_b^p, respectively, and assume that p_a^p is on Σ_1, while p_b^p is at a distance of 70,000 km from the Moon. A generic patching point p_τ^p along the EM trajectory can be parametrized by $\tau \in [0,1]$, so that its time of flight is given by $t_a^p + \tau \times (t_b^p - t_a^p)$, as illustrated in Fig. 3.

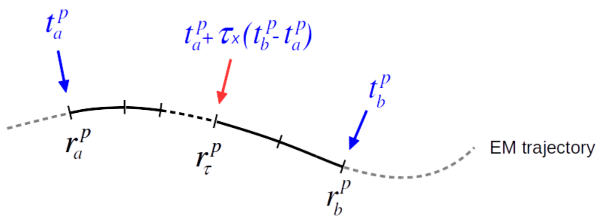

Figure 3: Parametrization of a patching point p_τ^p along the EM trajectory using the time of flight.

So, given one EM trajectory translated into the SE system, for fixed γ_0 and φ_1^0 defined accordingly, three parameters can be explored to produce optimized Earth-Moon transfer solutions, namely, $\delta\dot{x}$, $\delta\dot{y}$, and τ. Thus, the search for low-cost low-time solutions can be restated as a multiobjective optimisation problem in three design variables.

A number of sets of objective functions can be considered. Besides the manoeuvre at the patching point, an impulsive thrust of magnitude Δv_1 at the perigee is needed, such that, the total cost of an entire transfer is given by $\Delta v_t = \Delta v_1 + \Delta v_2$. Therefore, the functions to be minimised in this approach can be Δv_t, the altitude of perigee at departure (h_e), and also the total time of the transfer (t_{of}).

Genetic algorithms (GAs) are suited to solve the problem at hand. In particular, we employ the Nondominated Sorting Genetic Algorithm II (NSGA-II) [11], a fast and elitist multiobjective genetic algorithm capable of dealing with constrained problems.

Each run of NSGA-II requires one specific EM trajectory and provides one Pareto curve, with a number of SE solutions, corresponding to a number of trade-off possibilities. Relevant properties of the EM solutions, such as, t_{esc} and h_m, along with the trade-off options resulting from the optimization problem, can be organized into catalogues that allow to select full Earth-Moon transfers with specific profiles.

So, a systematic procedure to obtain ensembles of optimized Earth-Moon transfers using the patched three-body approximation with tilted planes can be summarized as: (i) perform a short-time analysis in the EM system to classify a large number of initial conditions into sets $\mathscr{G}$, $\mathscr{L}$, $\mathscr{H}$, $\mathscr{C}$, and $\mathscr{O}$; (ii) perform a long-time analysis in the EM system to select ballistic capture trajectories with adequate profiles from sets $\mathscr{G}$, $\mathscr{L}$; (iii) translate the candidate patching states to the SE system and set up a multiobjective optimisation problem in three design variables ($\delta\dot{x}$, $\delta\dot{y}$, and τ).

IV. RESULTS

The short-time analysis described in the previous section was applied to grids of initial conditions for 200 values of the Jacobi constant $C = C_i$, $i = 1, 2, \ldots, 200$, with $C_1 = 3.20034490 \leqslant C_i \leqslant C_{200} = 3.02043948$. The values of C_i were determined by the continuation method employed to compute 200 Lyapunov orbits around L_1^{EM}. The initial conditions were defined in Σ_1 related to the first Poincaré cut of W_o^s of $\Gamma(L_1^{EM})$.

Figure 4 shows a sample of the results of the short-time analysis. The plots show the initial conditions on the grid coloured according to the classification of the trajectories for four different values of C, namely, $C_{23} = 3.19444814$, $C_{30} = 3.19065379$, $C_{60} = 3.16800736$, and $C_{160} = 3.06453260$. The cut of W_o^s of $\Gamma(L_1^{EM})$ in Σ_1 is also shown as a black curve, and the white areas in frames (c) and (d) correspond to the forbidden regions at the given energy level. The plots illustrate how the spatial disposition of the sets changes as C varies and gives a visual indication of the amount of initial conditions in each set. To complement the visual information of Fig. 4, Table 1 gives additional information on the number of initial conditions in $\mathscr{G}$ that remain ballistically captured around the Moon for more than one month. The number shown in parenthesis on the first column corresponds to amount of trajectories that satisfy conditions (i) or (iii) of the subsequent long-time analysis over the points in set $\mathscr{G}$.

Table 1: Number of inital conditions in $\mathscr{G}$ with t_{esc} greater than 30, 60, 90, 120, and 180 days. # corresponds to amount of trajectories that satisfy conditions (i) or (iii) of the subsequent long-time analysis over the points in set $\mathscr{G}$.

index of	t_{esc} (days) greater than				
C (#)	30	60	90	120	180
23 (2780)	2780	9	2	0	0
30 (4248)	3633	931	156	26	4
60 (1865)	54	0	0	0	0
160 (263)	15	0	0	0	0

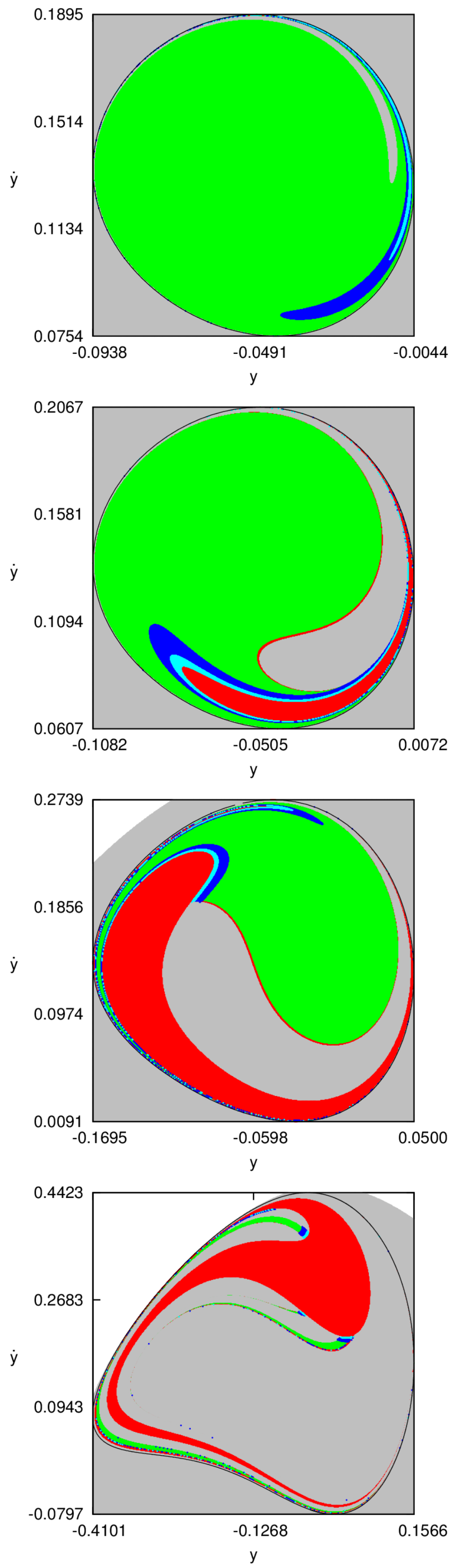

Figure 4: Sets $\mathscr{G}$ (blue), $\mathscr{L}$ (cyan), $\mathscr{C}$ (red), $\mathscr{H}$ (green), $\mathscr{O}$ (grey) in Σ_1 for four different values of C: (a) C_{23}, (b) C_{30}, (c) C_{60}, and (d) C_{160}.

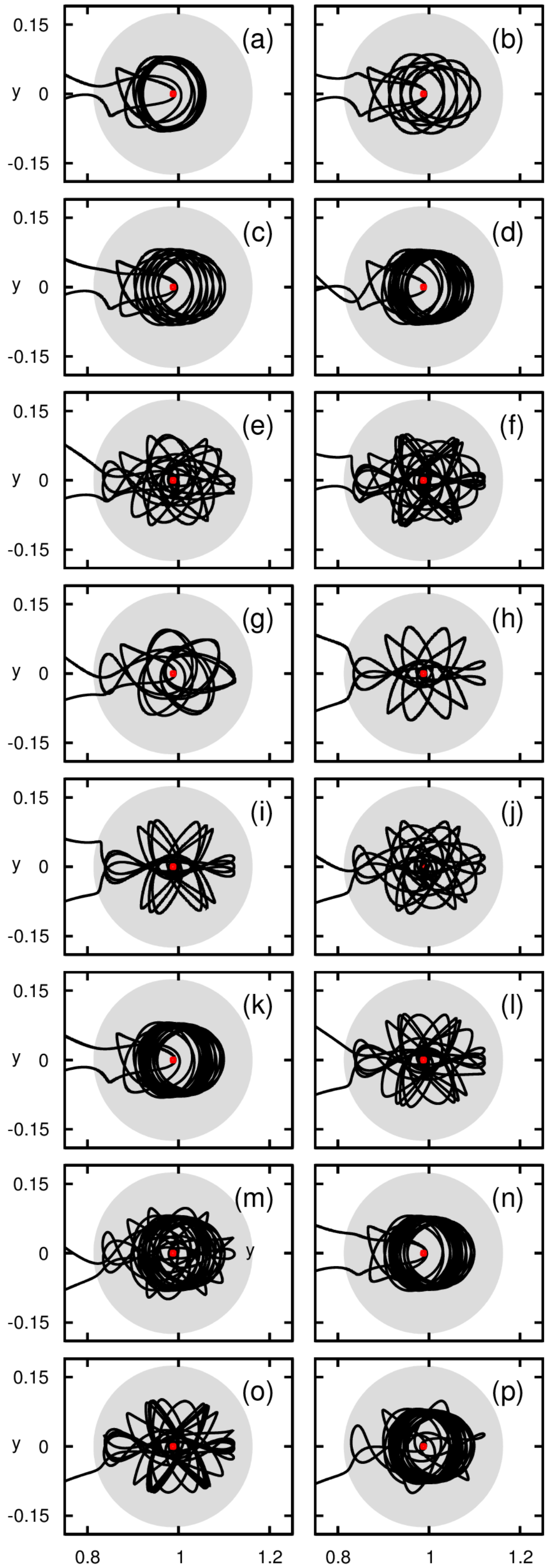

Figure 5: Capture trajectories in the EM reference frame.

For the 200 values of C analysed in [12], the values of C ranging from C_{25} to C_{36} have the best candidates to be selected for complete low-cost short-time Earth-to-Moon transfers, because they present the longest capture time, as well as the best profiles in the configuration space. In particular, the best results for long time capture are found for values of C around C_{30}. Figure 5 shows several capture trajectories with $C_{29} = 3.19123978$ and C_{30}. In the plots, the Moon is shown as a red circle, and the lunar SOI is depicted in grey. Table 2 presents some dynamical properties of these trajectories.

Table 2: Properties of the sample capture trajectories shown in Fig. 5.

	C	h_m (km)	t_{esc} (days)
(a)	29	153.25	89.02
(b)	29	116.65	62.57
(c)	29	192.52	86.38
(d)	29	123.03	111.37
(e)	29	156.09	107.23
(f)	29	96.28	168.82
(g)	30	334.03	80.68
(h)	30	220.69	62.35
(i)	30	362.86	90.68
(j)	30	239.60	102.46
(k)	30	118.58	159.76
(l)	30	182.00	121.55
(m)	30	376.91	164.50
(n)	30	328.61	over 172.43
(o)	30	172.48	over 172.81
(p)	30	118.09	over 172.35

Once a EM capture orbit is selected, the state vectors in the patching region are translated into the SE system synodic reference frame and the NSGA-II is used to explore quasi-periodic SE solutions to target the Earth.

Trajectory (f) of Fig. 5 is chosen to illustrate the optimization procedure, with Δv_t, h_e, and t_{of} taken as objective functions with h_e constrained. The design variables are $\delta\dot{x} \in [-0.06, 0.06]$, $\delta\dot{y} \in [-0.06, 0.06]$, and $\tau \in [0, 1]$. Two different constraints in the altitude of the perigee were explored.

- Case A: h_e is subject to the constraint
  $100\ \text{km} \leqslant h_e \leqslant 1{,}000\ \text{km}$
- Case B: h_e is subject to the constraint
  $35{,}000\ \text{km} \leqslant h_e \leqslant 100{,}000\ \text{km}$.

In both cases, the optimized solutions can be considered as initial guesses to compute fully optimized EM transfers with low, high, or hybrid thrust, and applying more realistic models [13]. The value of γ_0 was set to 1.9497 rad, which corresponds approximately to an epoch t_0 = 6504.3 MJD2000 (i.e., October 22, 2017, 19:11), with $\varphi_1^0 = 0.5163$ rad. Because both nodes result in the same cost and in the same time of flight, we have arbitrarily chosen to work with the descending node.

Figure 6 presents the Pareto fronts of both cases, illustrating the trade-off between Δv_t, t_{of} and h_e. The colour of the points is proportional to Δv_2 (km/s). Each point in the Pareto front corresponds to a complete Earth-Moon transfer solution. The ensembles of solutions generated after each run of the NSGA-II can be organized into a catalogue from which individual trajectories can be chosen to fulfill specific criteria.

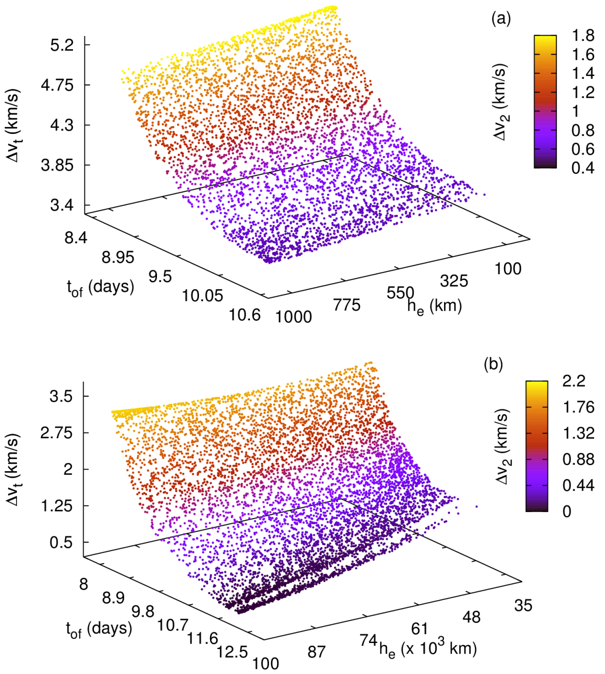

Figure 6: Pareto fronts obtained for the optimal problem with three objective functions for case (a) A and (b) B, presenting Δv_t (km/s) as a function of h_e (km) and t_{of} (days).

Figures 7 and 8 present optimal full EM transfers, both in the 3D EM synodical reference frame and 2D SE synodical reference frame. While Fig. 7 refers to case A, Fig. 8 refers to case B. The altitude of the perigee was the first criterion used to select three trajectories for each case, namely low, mid, and high altitude inside the permitted range, labeled (f.X1), (f.X2), and (f.X3), respectively, with 'X' replaced by 'A' or 'B' according to the case. Then, t_{of} was chosen so that the transfers had the smallest values of Δv_t in that range of h_e. The EM capture orbit is the same for all the exemples, namely, the trajectory in Fig. 5 (f). In each case, the patching point for is different according to the solution of the optimization algorithm.

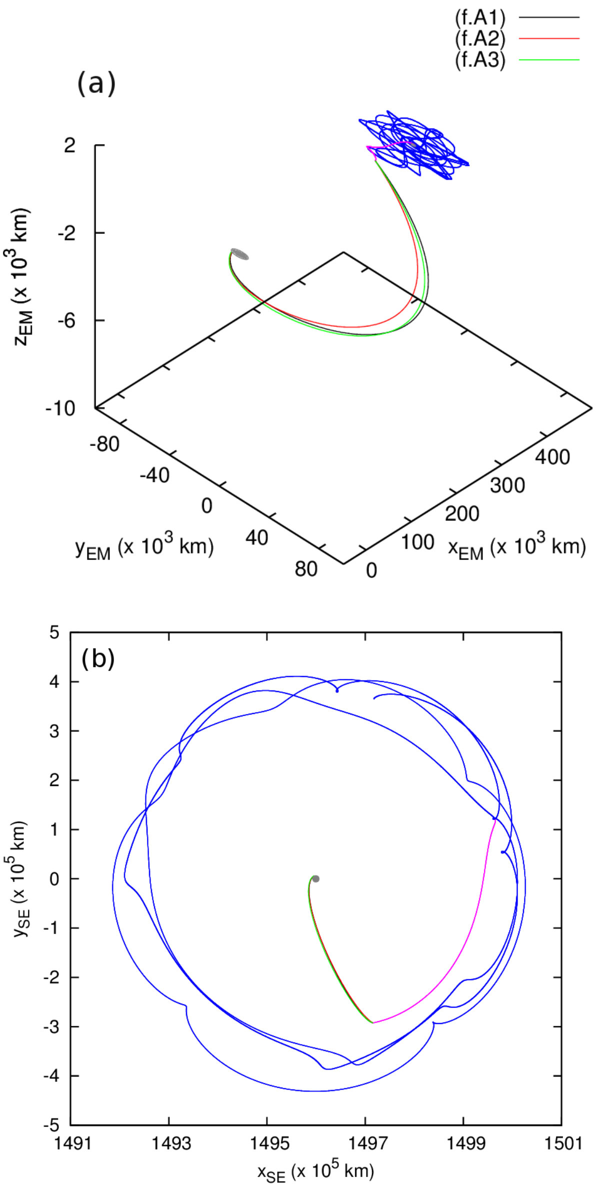

Figure 7: Full short-time low-cost Earth-Moon transfers for case A in the (a) EM and (b) SE synodical reference frames.

Tables 3 and 4 present relevant properties of these solutions. In all the examples, the t_{of} varies from 9 to 13 days. In case A, the trade-off between time of flight and total cost is quite reasonable when compared to classical two-impulsive Earth-Moon transfers based on many-body dynamics found in the literature, so the Earth-Moon transfers with ballistic capture at the arrival are interesting alternative solutions in this context. On the other hand, trajectories in case B are more likely to be provide feasible solutions when used as initial guesses to search optimal transfers in more realistic models and considering different propulsion technologies, because they require realistic nominal values of the maximum low-thrust acceleration.

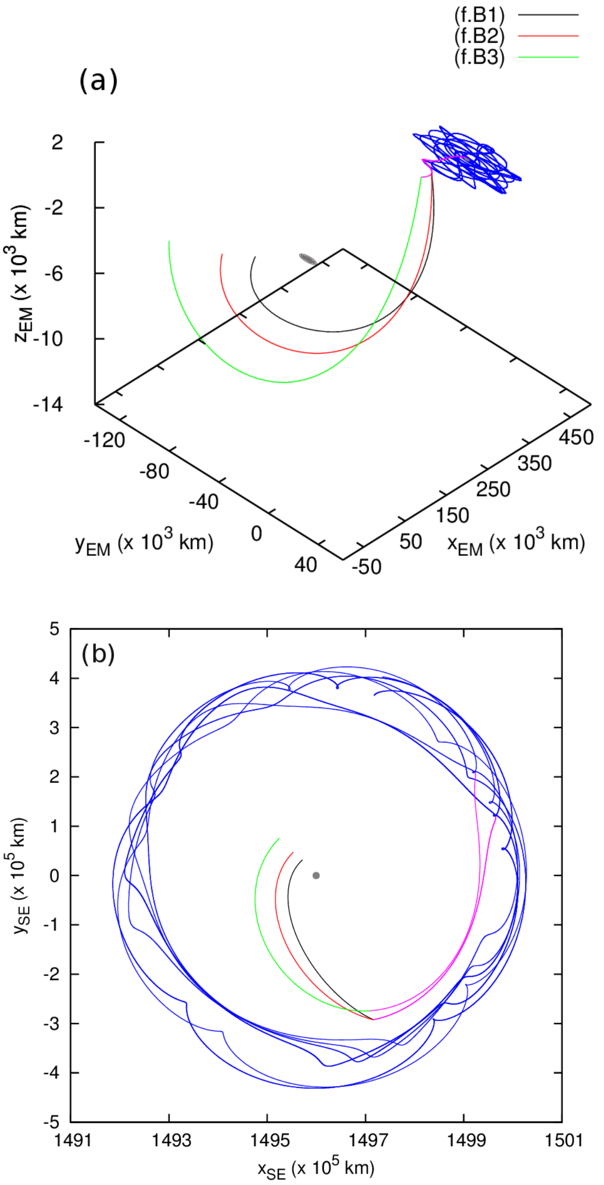

Figure 8: Full short-time low-cost Earth-Moon transfers for case B in the (a) EM and (b) SE synodical reference frames.

Table 3: Properties of the full transfers in case A shown in Fig. 7.

	h_e (km)	Δv_t (km/s)	Δv_2 (km/s)	t_{of} (days)	t_{SE} (days)
(f.A1)	$\approx$ 190	3.7199	0.5872	10.38	3.37
(f.A2)	$\approx$ 600	3.6185	0.5847	10.16	3.15
(f.A3)	$\approx$ 1,000	3.5166	0.5737	10.36	3.34

Table 4: Properties of the full transfers in case B shown in Fig. 8.

	h_e (km)	Δv_t (km/s)	Δv_2 (km/s)	t_{of} (days)	t_{SE} (days)
(f.B1)	$\approx$ 35,786	1.5125	0.4650	9.96	2.94
(f.B2)	$\approx$ 60,000	1.0675	0.3338	10.37	3.36
(f.B3)	$\approx$ 100,000	0.5071	0.0779	12.22	4.04

V. FINAL REMARKS

We have discussed a heuristic strategy to obtain ensembles Earth-Moon transfers with ballistic capture using a patched three-body approximation, that is, a decomposition of the Sun-Earth-Moon-Spacecraft system into two planar Circular Restricted Three-Body Problems, namely, the EM system and the SE system. A more realistic model was introduced by taking into account the inclination of the orbital plane of the Moon with respect to the ecliptic.

The methodology includes an extensive analysis to look for ballistic capture trajectories in the EM system. Then, the capture solution are translated into the SE system to investigate the patching possibilities and the cost of targeting the Earth backwards using quasi-periodic solutions of the SE system. For this, a genetic algorithm is employed, so for every ballistic capture orbit of the EM system, a population of full transfers is obtained. The transfers are Pareto optimal solutions with respect to the total transfer time, the total Δv, and the altitude at departure.

The full solutions are an interesting alternative to classical two-impulsive Earth-Moon transfers based on many-body dynamics. Moreover, they can be used as initial guesses to compute fully optimized solutions considering different propulsion technologies, such as low, high, and hybrid thrust, and/or using more realistic models.

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